Type-2 Fuzzy Control Lyapunov Approach for Position Trajectory Tracking

R. Farfan-Martinez[1], J.A. Ruz-Hernandez[2], J.L. Rullan-Lara[2], W. Torres-Hernandez[1], and L.A. Cambrano-Bravata[1]

[1]Universidad Tecnologica de Campeche. Carretera Federal 180 s/n, San Antonio Cardenas, C.P. 24381, Carmen, Campeche. Mexico.
Tel: 01 (938) 3816700, Ext. 120
{farfan678,williantorreshernandez,luzbra}@hotmail.com
[2]Universidad Autonoma del Carmen. Calle 56, No. 4 Esq. Avenida Concordia, Col. Benito Juarez, C.P. 24180. Ciudad del Carmen, Campeche, Mexico.
{jruz,jrullan}@pampano.unacar.mx@hotmail.com
Paper received on 12/10/13, Accepted on 01/19/14.

Abstract. The paper presents the design of type-2 fuzzy controller using the *fuzzy Lyapunov synthesis approach* in order to systematically generate the rule base. To construct the rule base, the error signal and the derivative of the error signal are considered. It also presents the performance analysis to determine the value of the separation interval ξ between the upper and lower membership functions of the type-2 fuzzy set used. The controller is implemented via simulation to solve trajectory tracking problem for angular position of a servo trainer equipment in presence of backlash. Simulation results are successful and show better performance than a classic controller.

Keywords: Fuzzy Control, Lyapunov Approach, Nonlinearities, Servo Trainer.

1 Introduction

The fuzzy sets were introduced by L. A. Zadeh in the mid-sixties in order to process data affected by non-probabilistic uncertainty [1]. The type-1 fuzzy systems can handle the linguistic variables and experts reasoning and also reproduce the knowledge of systems to control, however, it can not handle uncertainties such as dispersions in linguistic distortion measurements and expert knowledge [2]. On the other hand, type-2 fuzzy systems can handle such kinds of uncertainties and also have the ability to model complex nonlinear systems. In addition, controllers designed using type-2 fuzzy systems achieve better performance than those of type-1. The type-2 fuzzy sets were also originally proposed by Zadeh in 1975 [3].

In [4] a fuzzy logic type-2 based controller using genetic algorithms is performed to control the shaft speed of a DC motor. Genetic algorithms are used to optimize triangular and trapezoidal membership functions. The controller is implemented in a FPGA and its performance is compared with fuzzy logic type-1 and PID controllers.

A type-2 fuzzy controller (T2FC) is designed for an automatic guided vehicle for wall-following in [5]. In this case, T2FC has more robustness to sensor noise and better guidance performance than one of type-1. Another application of T2FC to mobile robots is presented in [6]. Trajectory tracking is applied first at simulation level and then on a Digital Signal Controller (DSC) of a experimental platform. The reported results show that performance of type-1 controller is poor comparing to type-2 controller.

Some applications of type-2 fuzzy controller in real-time can be also found in literature. For example, the classical inverted pendulum and the magnetic levitation system which are both highly non-linear. In [7], a low-cost microcontroller is used to validate the performance of T2FC for the inverted pendulum. For magnetic levitation system, [8] compared performance of type-1 and type-2 fuzzy controllers and a PID controller. Given that the system is unstable and non-linear, T2FC is showed better performance. Finally, position and velocity type-1 controller are designed in [9]. In this case, stability of both controllers are assured by means of Fuzzy Lyapunov Approach [10]. Results are presented in real time and are compared with classic controllers.

The paper is organized as follows. In section 2 we describe the servo trainer equipment and the characteristics of backlash. Then, section 3 presents the control design methodology using the fuzzy Lyapunov approach. Simulation results are presented in section 4. Finally, concluding remarks are presented in section 5.

2 Servo Trainer Equipment

The equipment used as plant to control in this paper is the *CE110 Servo Trainer* from *TQ Education and Training Ltd* [11]. This apparatus is used to help in teaching linear control theory and to implement validate some control algorithms (classical and non classical) in real-time.

The equipment have a variable load which is set using a current direct generator, by changes of different inertial load and using the engage a gearbox or by set all of them together. Besides, the apparatus have three modules to introduce some typical nonlinearities.

The mathematical model of servotrainer is set by equations [9]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\tfrac{1}{T}x_2 + \tfrac{G_1 G_2}{T}u \end{aligned} \tag{1}$$

where $x_1 = \theta$ and $x_2 = \omega$ are the angular position and angular velocity, respectively. The gains G_1 and G_2 are defined by $G_1 = k_i k_\omega$ and $G_2 = k_\theta/30k_\omega$ where $k_i = 3.229$ (rev/sec-Volts) is the motor constant, $k_\omega = 0.3$ (Volts/(rev/sec)) is the velocity sensor constant and $k_\omega = 20$ (Volts/rev) is the angle sensor constant. The time constant T change according to size of load: $T = 1.5$ (sec) for small load (one inertial disc); $T = 1$ (sec) for medium load

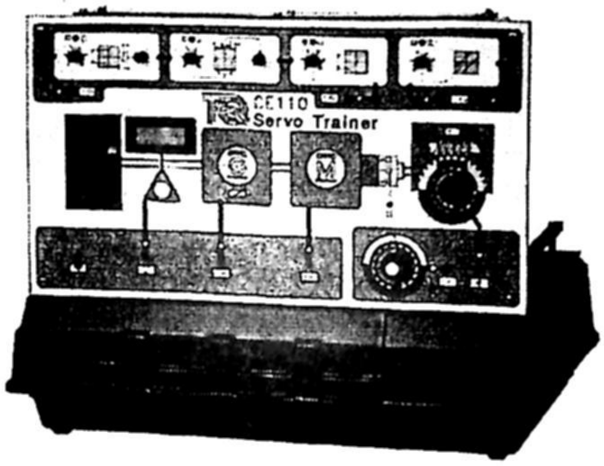

Fig. 1. Servo Trainer

(two inertial discs); $T = 0.5$ (sec) for large load (three inertial discs). (see Fig. 1).

The equipment provides a hysteresis block, to simulate and study the important feature of backlash in the use of the gearbox and the effect on Servo Trainer.

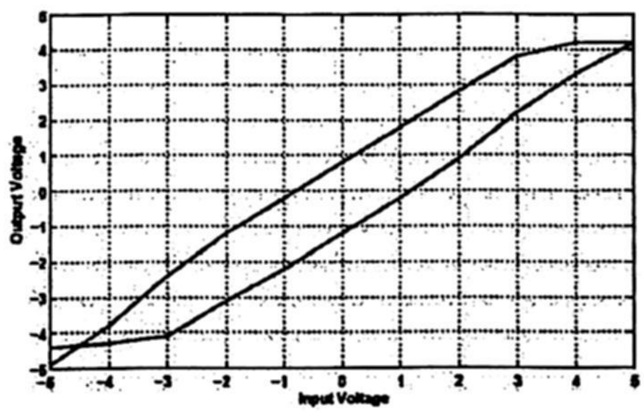

Fig. 2. Hysteresis Characteristics

The characteristics of the Servo Trainer hysteresis block are calculated experimentally (see Fig. 2) with width of 1 volt. Input and output voltage are taken in the block to determine its characteristics.

3 Controller Design

3.1 Fuzzy Lyapunov Approach

The goal is to design a control law u such that the velocity and position of servo trainer follows a reference signal y_{ref}. One way of achieving this goal is to choose a Lyapunov function candidate $V(x)$. Then, this Lyapunov function must meet the following requirements [10]:

$$V(0) = 0 \ , \tag{2}$$

$$V(x) > 0, \ \ x \in N\backslash\{0\}, \tag{3}$$

$$\dot{V}(x) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i}\dot{x}_i < 0, \ \ x \in N\backslash\{0\}. \tag{4}$$

where $N\backslash\{0\} \in R^n$ is some neighborhood of $\{0\}$ excluding the origin $\{0\}$ itself, and $\dot{x}_i$ $(i = 1, 2, \ldots, n)$. If $\{0\}$ is an equilibrium point of (1) and such $V(x)$ exist, then $\{0\}$ is locally asymptotically stable.

The conditions (2) and (3) are satisfied by taking such Lyapunov function candidate $V = \frac{1}{2}\left(e^2 + \dot{e}^2\right)$ where e is the tracking error. Differentiating V we have $\dot{V} = e\dot{e} + \dot{e}\ddot{e}$. Substituting $w = \ddot{e}$, is required then:

$$\dot{V} = e\dot{e} + \dot{e}w < 0 \tag{5}$$

Analyzing the equation (5), we can establish four basic fuzzy rules for w such that conditions (4) is satisfied:

- IF e is *positive* AND $\dot{e}$ is *positive* THEN w is *negative big*
- IF e is *negative* AND $\dot{e}$ is *negative* THEN w is *positive big*
- IF e is *positive* AND $\dot{e}$ is *negative* THEN w is *zero*
- IF e is *negative* AND $\dot{e}$ is *positive* THEN w is *zero*

3.2 Type-2 Fuzzy Systems

A fuzzy type-2 system denoted by $^{\approx}A$, is characterized by a membership function type-2 $\mu_{\approx A} = (x, u)$, where $x \in X$, $u \in J_x^u \subseteq [0,1]$ and $0 < \mu_{\approx A} = (x, u) < 1$. It is defined as follows [12]

$$^{\approx}A = \{(x, \mu_A(x) \mid x \in X)\} = \left[\int_{x\in X} \left[\int_{u\in J_x^u \subseteq [0,1]} f_x(u)/u\right]/x\right] \tag{6}$$

If $f_x(u) = 1$, $\forall u \in [\underline{J_x^u}, \overline{J_x^u}] \subseteq [0,1]$, membership function type-2 $\mu_{\approx A}$ is expressed by a lower membership function type-1 $\underline{J_x^u} = \underline{\mu}_A(x)$ and upper membership function type-1 $\overline{J_x^u} = \overline{\mu}_A(x)$. Then, $\mu_{\approx A}$ is called an fuzzy type-2 interval, denoted by equation (7)

$$^{\approx}A = \left[\int_{x\in X} \left[\int_{u\in[\underline{\mu}_A(x), \overline{\mu}_A(x)]\subseteq[0,1]} 1/u\right]/x\right] \tag{7}$$

If $^{\approx}A$ is a fuzzy type-2 singleton, then the membership function is defined by equation (8)

$$\mu_{\approx A}(x) = \begin{cases} 1/1, & \text{if } x = x' \\ 1/0, & \text{if} x \neq x' \end{cases} \tag{8}$$

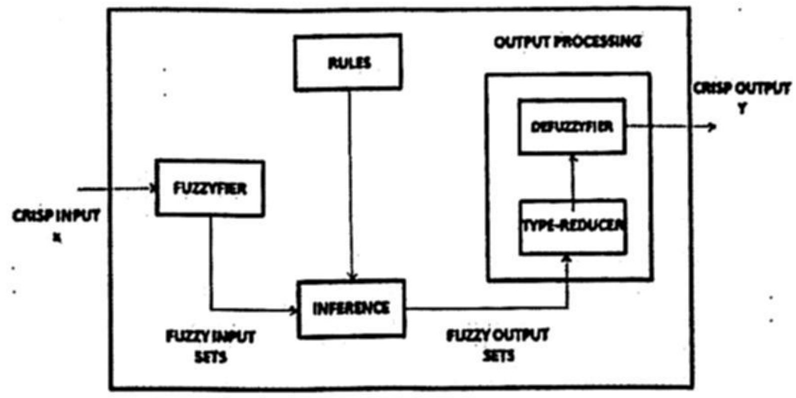

Fig. 3. Components of a type-2 fuzzy system

The type-2 fuzzy systems consist of a fuzzyfier which converts a value from real world into a fuzzy value, a fuzzy inference engine that applies a fuzzy reasoning to obtain a fuzzy output, an output processor comprising a reducer that transforms a fuzzy set type-2 into a fuzzy set type-1 and defuzzyfier which converts a fuzzy value into a precise value (see Fig. 3).

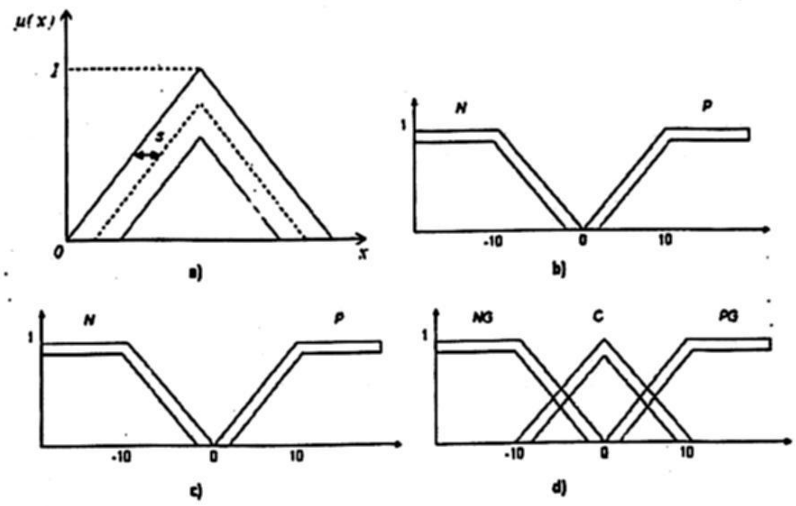

Fig. 4. Type-2 fuzzy sets: (a) Definition of type-2 fuzzy set; (b) Fuzzy set for e; (c) Fuzzy set for $\dot{e}$; (d) Fuzzy set for w

As mentioned above, membership functions in type-2 fuzzy systems are characterized by having two membership functions of type-1; an upper and a lower membership function. The interval ξ between these two functions can be varied in order to obtain optimal performance [13]. Figure 4a shows such type-2 membership function.

In this paper we have used the Matlab Toolbox developed and described in [12] to implement the type-2 fuzzy system in order to generate values of w. Figures 4b-c shows fuzzy sets for error e, for the derivative of error $\dot{e}$ and for variable w, respectively.

3.3 Mamdani Position Controller

The goal is to design a control signal u such that the angular position x_1 follows a desired reference signal y_θ. That is, $e_\theta \to 0$ as $t \to \infty$ where $e_\theta = x_1 - y_\theta$. In this case, $\ddot{e}_\theta$ is related to w by $\ddot{e}_\theta = w = \ddot{x}_1 - \ddot{y}_\theta$. From equation (1), we have that $\ddot{x}_1 = \dot{x}_2$ and the expression for w is $w_\theta = -\frac{1}{T}x_2 + \frac{G_1G_2}{T}u - \ddot{y}_\theta$. Then, the control signal u for position tracking is

$$u = \frac{T}{G_1G_2}(w_\theta + \ddot{y}_\theta) + \frac{1}{G_1G_2}x_2 \tag{9}$$

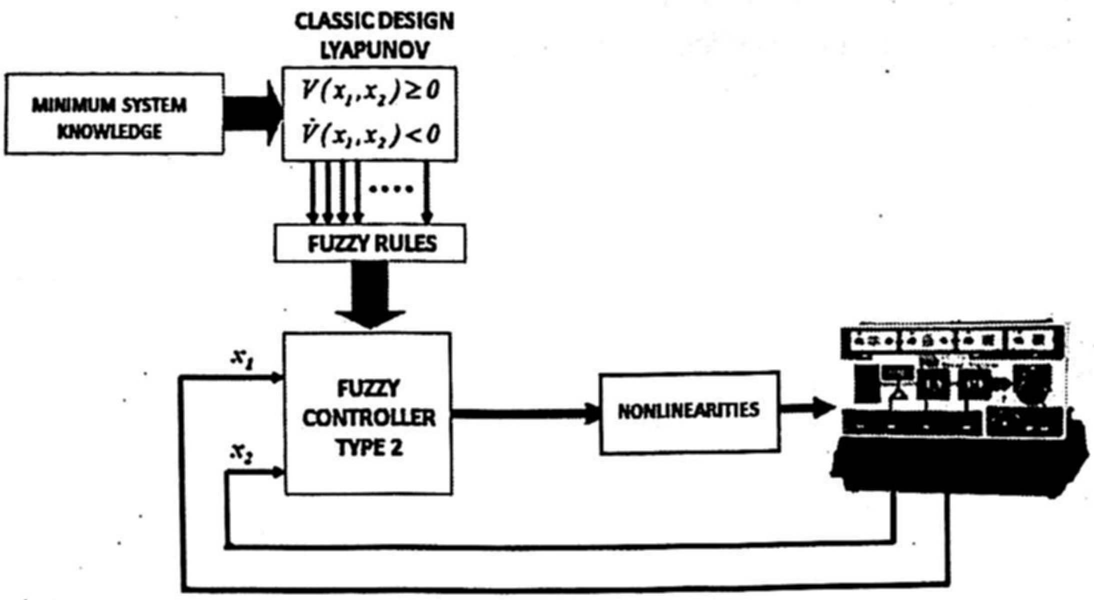

Fig. 5. Control scheme

Figure 5 shows the control scheme used for the simulations. The mathematical model of Servo Trainer and hysteresis block characteristics are determined experimentally in the equipment.

4 Simulation Results

In this section the *integral of the absolute value of the error* (IAE) and the *integral square error* (ISE) are used as performance criteria of proposed controller.

4.1 Position controller

The reference signal is the sine signal $y_\theta = 108 \sin 0.3t$ degrees and small load conditions are considered. Parameter ξ for type-2 fuzzy sets is set to 0.1. Performance of our controller is compared to those of a classical controller with a proportional controller for x_1 ($k_p = 10$) combined with a velocity feedback loop gain with $k_v = 0.01$ [11].

In Figure 6(Left) we can observe the trajectory tracking. Both controllers have acceptable performance in tracking the position trajectory, but the classic controller has oscillations in the output from the start and kept until the end of the simulation. Looking the tracking errors (Up-Right side of Fig. 6) shows that the error T2FC is smaller in magnitude than the classic controller error. It is also observed that both control signals have oscillations during the simulation, but the control signal T2FC presents smaller variations. Both signals are within the limits of the actuator of the equipment.

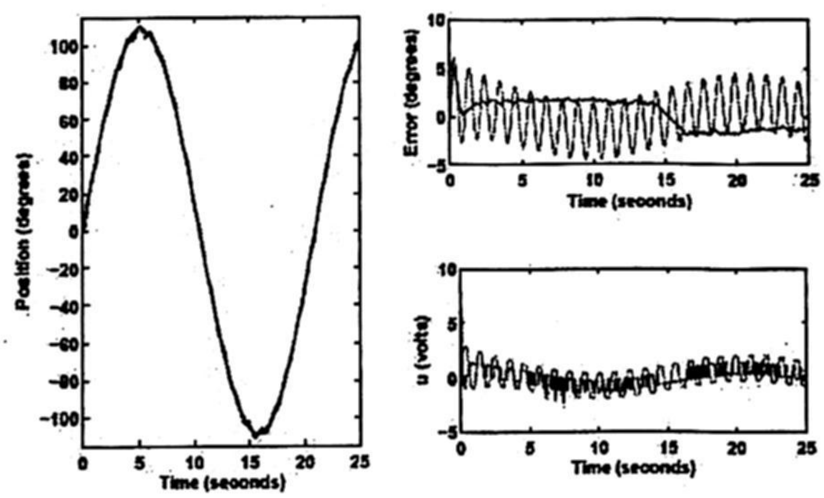

Fig. 6. Position Tracking y_θ (blue line), T2FC (green line), classical controller (red line). Left: Trajectory tracking; Up-Right: error signal; Down-Righ: control signal

Table 1. Performance of trajectory tracking

	Controller	IAE	ISE
Position Tracking	Type-2 ($\xi = 0.1$)	2.06	0.1818
	PI ($k_p = 10$, $k_i = 0.01$)	2.919	0.4669

Finally, the Table 1 shows the performance in terms of IAE and ISE error criterions. Our type-2 fuzzy controllers had proved a good performance of the proposed approach and surpass performance of classical controller.

5 Conclusions

In this paper we have design type-2 fuzzy controller using the *fuzzy Lyapunov synthesis approach* in order to systematically generate the rule base. Controller is designed to solve the position trajectory tracking problem in a servo trainer system. To tuning the type-2 fuzzy controller, the separation between upper and lower membership functions is commanded by parameter ξ in steps of 0.1 units. The best tuning was obtained with $\xi = 0.1$.

The performance of our proposed controller is compared to classical controller under same simulations conditions for the servo trainer. The IAE and *ISE* are used as performance criterions. Simulation results had proved good performances of our proposed approach in position tracking applications and surpass performances of classical controller.

Actual research is conducted to test our controllers in medium and full load conditions in the servo trainer equipment. In order to demostrate the effectiveness of our approach, authors are motivated to compare performances of type-2 fuzzy controller with performance of type-1 fuzzy controller.

6 Acknowledgments

The first and the fourth author thank to Universidad Tecnologica de Campeche for financial support and to Faculty of Engineering of the Universidad Autonoma del Carmen for providing the facilities for the use of equipment used in this work.

References

1. Zadeh, L., A. Fuzzy Sets. Information and control 8, 338–353 (1965)
2. Kwak, H.J., Kim, D.W., Park, G.T., A New Fuzzy Inference Technique for Singleton Type-2 Fuzzy Logic Systems. International Journal of Advanced Robotic Systems 9, 1–7 (2012)
3. Zadeh, L.,A. The concept of a linguistic variable and its application to approximate reasoning-1, Informat. Sci. 8, 199–249 (1975)
4. Maldonado, Y., Castillo, O. Genetic Design of an Interval Type-2 Fuzzy Controller for Velocity Regulation in a DC Motor. International Journal of Advanced Robotic Systems 9, 1–8 (2012)
5. Yao, L., Chen, Y.S., Type-2 Fuzzy Control of an Automatic Guided Vehicle for Wall-Following. In: Lucian Grigorie (ed.), Theory and Applications, pp. 243-252, ISBN: 978-953-307-543-3 (2011)
6. Leottau, L., Melgarejo, M. An Embedded Type-2 Fuzzy Controller for a Mobile Robot Application. In: Andon Topalov (Ed.), Recent Advances in Mobile Robotics, pp. 365–384, ISBN: 978-953-307-909-7. (2011)
7. Sierra, G.K., Bulla, J.O., Melgarejo, M.A., An Embedded Type-2 Fuzzy Processor For The Inverted Pendulum Control Problem. IEEE Latin America Transactions 9(3), 263–269 (2011)
8. Kumar, A., Kumar, V., Design and Implementation of IT2FLC for Magnetic Levitation System. Advances in Electrical Engineering Systems 1 (2), 116–123 (2012)

9. Ruz, J.A., Rullan, J.L., Garcia, R., Reyes, E.A., Sanchez, E. Trajectory Tracking Using Fuzzy-Lyapunov Approach: Application to a Servo Trainer. In: Castillo, O., Melin, P., Montiel Ross, O., Sepulveda Cruz, R., Pedrycz, W., Kacprzyk, J. (Eds.), Theoretical Advances and Applications of Fuzzy Logic and Soft Computing, Springer-Verlag, pp. 710-718 (2007)
10. Margaliot, M., Langholz, G., Fuzzy Lyapunov-based approach to the design of fuzzy Controllers. Fuzzy Sets and Systems, Elsevier 106, 49–59 (1999)
11. TecQuipment LTD: CE110 Servo Trainer, User's Manual, England (1993)
12. Castro, J.R., Castilo, O., Melin, P., Martinez, L. G., Escobar, S., Camacho, I., Building Fuzzy Inference Systems with the Interval Type-2 Fuzzy Logic Toolbox. In Melin, P. et al. (Eds.), Analysis and Design of Intelligent Systems using Soft Computing Techniques, Springer-Verlag, pp. 53–62 (2007)
13. Cazarez N. R., Castillo O, Aguilar L. and Cardenas S.: Lyapunov Stability on Type-2 Fuzzy Logic Control, Proceedings of International Seminar on Computational Intelligence, Mexico D. F., pp. 32-41, (2005)